REVIEW

On the thesis of _____

Ostroukhov Petr Alexeevich

High-order methods for optimization problems with specific structure

Submitted for the degree of candidate of <u>physical and mathematical</u> sciences in specialisation <u>1.2.3</u>. Theoretical Informatics, Cybernetics.

Author of the review

Name: Raigorodskii Andrey Mikhailovich

Degree: Doctor in Physics and Mathematics

The year of awarding the degree and the scientific speciality in which the degree is awarded: 2004, 01.01.09. Discrete mathematics and mathematical cybernetics

Place of work: Moscow Institute of Physics and Technology

Position: Director of the Phystech School of Applied Mathematics and Informatics

Contact information: mraigor@yandex.ru

The first chapter introduces two innovative approaches for tackling convex optimization problems characterized by linear equality constraints and dual objectives possessing Lipschitz p-th order derivatives. The first method focuses on minimizing the gradient's norm of the dual problem and subsequent reconstructing an approximate primal solution. Impressively, this work presents two near-optimal solutions, demonstrating the effectiveness of the approach. The application of these findings to the primal-dual setting showcases the versatility of the work. The second approach offers a direct accelerated primal-dual tensor method tailored to convex problems with linear equality constraints, where the dual objective exhibits Lipschitz p-th order derivatives. Proposed algorithms are validated through practical experiments, emphasizing their utility in scenarios such as logistic regression, entropy-regularized optimal transport problems, and minimal mutual information problems.

The second chapter introduces innovative approaches to address two types of saddle point problems, leveraging assumptions about objective function properties. It presents two algorithms for min-max problems, one with linear complexity and another with global linear convergence and local quadratic convergence, showcasing their performance optimization. Additionally, the adaptation of a convex optimization framework for gradient norm minimization underscores the versatility of these methods, making them valuable across various optimization scenarios.

The fourth chapter of this work dives into min-min problems, which arise when dealing with convex optimization involving groups of variables of different dimensions or domains. While prior articles primarily focused on zeroth and first-order oracles, this paper looks at application of high-order methods for inner problems and fast gradient methods for outer problems. Under various assumptions, including convexity, Lipschitz continuity, strong convexity, and compact constraints, the proposed methods demonstrate superior convergence rates compared to conventional approaches.

The final chapter explores Projection-free optimization using various adaptations of the Frank-Wolfe, or Conditional Gradient method, an important approach in machine learning optimization due to its cost-effective linear minimization oracle. They address scenarios where self-concordant functions with unbounded curvature pose challenges, and existing methods lack theoretical guarantees. Leveraging self-concordant function theory, this work introduces adaptive step sizes for Frank-Wolfe methods, demonstrating global sublinear convergence, and even achieve linear convergence rates when a stronger local linear minimization oracle is available. This chapter presents valuable advancements in optimization techniques with broad applicability in machine learning and other domains.

This thesis is a commendable contribution to the field of optimization, offering a comprehensive exploration of advanced techniques and their applications. The chapters delve into various aspects of convex optimization, saddle point problems, and projection-free optimization, demonstrating a deep understanding of theoretical foundations and practical considerations. Notably, the incorporation of high-order methods and the adaptation of Frank-Wolfe methods for machine learning contexts showcase the author's innovative approach to addressing real-world challenges. Furthermore, the ability to provide theoretical guarantees and improve convergence rates in complex scenarios highlights the thesis's significance and its potential impact on the optimization community. Overall, this work stands as a valuable resource for researchers and practitioners seeking advanced optimization solutions.

Date 25,09.23

/ A.M. Raigorodskii

